Phi

1.618

Recommended Reading

- The Golden Ratio: The Story of Phi, the World's Most Astonishing Number by Mario Livio - Written in 2002; Dr. Livio is the head of the Science Division at the Space Telescope Science Institute focusing on cosmology and astrophysics
- **Roger Herz-Fischler, A Mathematical History of the Golden Number**
- **Euclid's Elements**
- **Pythagoras and the Pythagoreans: A Brief History by Charles Kahn**
- **Measuring Heaven, Christiane Joost-Gaugier**

Great Quotations

- Edna St. Vincent Millay wrote a poem "Euclid Alone Has Looked on Beauty Bare"

- Buddha's sermon in which he simply held up a rose and did not utter a single word.

- Leopold Kronecker (19th century mathematician) said "God created the natural numbers, all else is the work of man."

- "Let no one ignorant of geometry enter" or "Let no one destitute of geometry enter my doors." Tradition has it that this phrase was engraved at the door of Plato's Academy, the school he had founded in Athens.

Linguistic and Anthropological Origins

- early humans - practical needs

- early distinctions one, two, many; counting systems from 5 millenia ago

- probably influenced by our bodies, also innate recognition of duality (sun/moon, day/night, male/female)

- two-counting - in early Sumeria, Zulu, Brazil - urapun, okosa, okosa-urupun, okosa-okosa

- why stop at 4 - physiology of visual perception - largest number we can capture in a glance is 4 or 5

What is Phi?

- An irrational number, i.e., a number that can't be expressed as a whole number of as the ratio of two whole numbers (rational numbers)

- 1.6180339887 (never-ending and never-repeating)

- In the 19th century, it was called "Golden Number", "Golden Ratio", and "Golden Section"

- Euclid referred to it as the division of a line into its "extreme and mean ratio"

- In professional mathematical literature, the Greek letter "tau" is used. Tau comes from tome, meaning, the cut or section. It was later given the name "phi" in honor of Greek sculptor Phidias, Parthenon sculptor

- Found in natural phenomena as well as art

- incommensurability

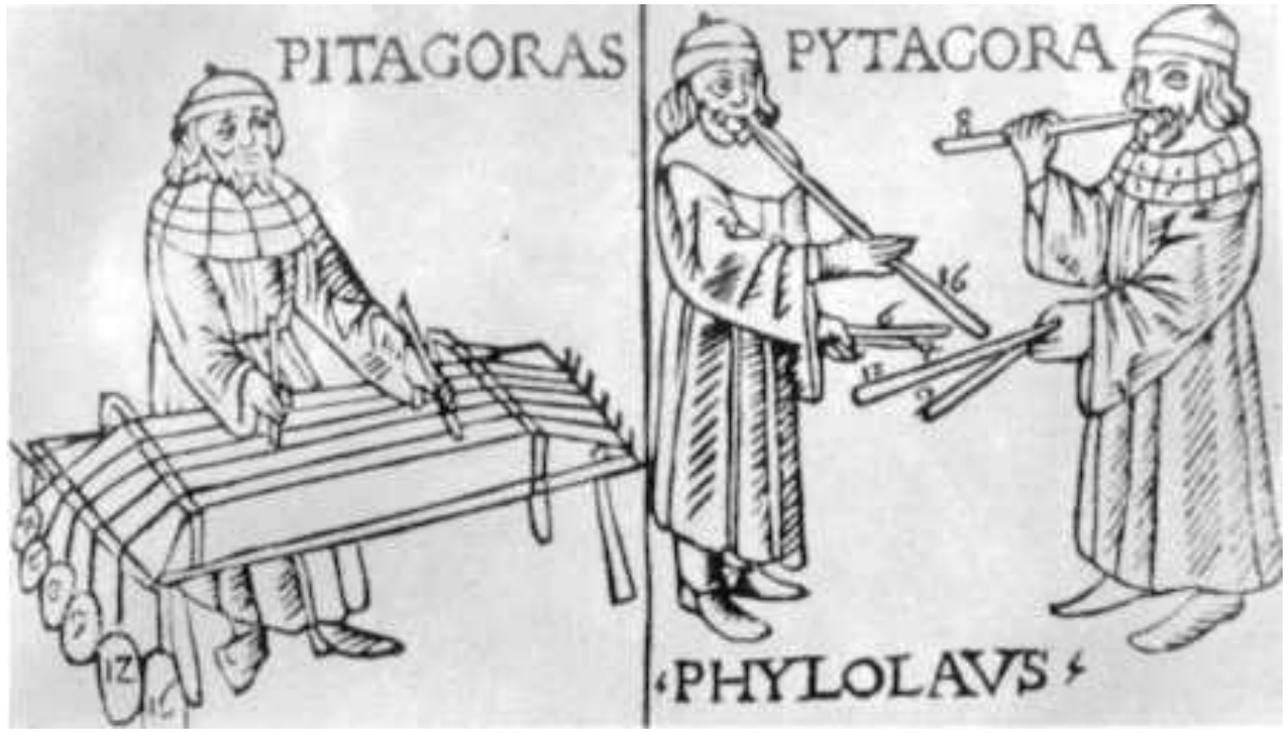
Definition of Phi and some pictures

- Line ratios
- Pentagram
- Geometric construction

Numerology and Gematria

- Mystical associations with number

Pythagoras the Musician



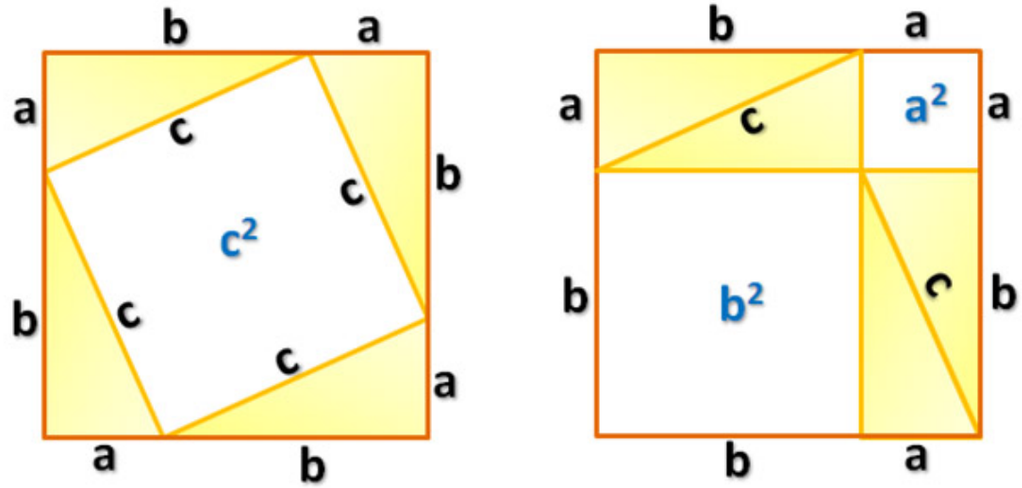
Pythagoras

- Born around 570 BC in the island of Samos (metaphysical movie on YouTube)
- Probably learned mathematics in Egypt. Contemporary of Buddha and Confucius
- Mingling of mathematics, philosophy, and religion. Coined the word "philosophy", love of wisdom
- Emphasized the importance of learning. mathetes - to learn - mathematics = "that which is learned"
- Pythagoras was one of the first to not just use numbers as practical tools but as abstract entities which exist in their own right. The basic forces of the universe may be expressed through the language of mathematics
- Believed in metempsychosis, the soul is immortal and reborn or transmigrated in human and animal incarnations
- Purify the soul through disciplines, vegetarianism
- Ancient Babylonians and Egyptians already knew about Pythagorean triples but not till Euclid was there proof
- Although there is mystical appeal in Golden Numberism, it is unlikely that the ancient Babylonians had discovered phi; also unlikely that the ancient Egyptians used phi in the design of their pyramids. The Greeks discovered it

Pythagorean Theorem

- a2+b2=c2

- Babylonians - Pythagorean triples



Pythagoras and Harmonics

- Tetraktys
- 2:3 fifth 3:4 fourth
- monochord
- Theorica Musice 1492
- music of the spheres - Johannes Kepler

Number

- The number 1 is seen as the generator of all other numbers and as such is not regarded as a number in itself. - unity- expressed with a point
- The number 2 is the division, duality - expressed with a line
- The number 3 - the first real number - combines unity and division (1 and 2)
- The number 4 - number of justice and order - the four directions - first number that connotes volume - 4 points not in the same plan form a tetrahedron - Pythagoreans liked 4 because 1 + 2 + 3 + 4 = 10, combines uniqueness, polarity, harmony, and space and matter
- 5 - Pythagoreans used the pentagram as the symbol of their brotherhood and they called it "health" - the pentagram has a relationship between its progressively smaller sides of phi, initial interest in phi probably stemmed from
- 6 is the first perfect number = the sum of all the smaller numbers that divide into it (1 + 2 + 3) - Augustine thought it was a perfect number

The Number 5, the Pentagram and Incommensurability

- The relation of a side to the diagonal in a pentagram is incommensurable, that is the ratio of their lengths can't be expressed as 2 rational numbers. (reductio ad absurdum proof)

- Ratio of segments = phi in progressively smaller pentagrams

Early history of the pentagram and phi

- 4000 BCE Mesopotamia - pentagram shapes found in Uruk. Also in Israeli and other excavations

- main question is what did they understand about the geometric properties of pentagrams

- Babylonians seem to have known about pi but no evidence that they knew about phi

- all the efforts to show their buildings were built with phi in mind are very inconclusive

- Same with Egypt, with the buildings of tombs and pyramids

- although it has mystical appeal, there is no solid evidence

- the task of discovering phi was left to the Greeks

Plato and Geometry

- Plato and his followers saw the sheer beauty in the solids; urged focusing on the abstract heaven of mathematics rather than astronomy

- Theaetetus - the first to construct the five Platonic solids - the only existing solids in which all the faces are identical and equilateral and each of the solids can be circumscribed by a sphere with all its vertices lying on the sphere

- According to Plato, each of the solids was associated with the elements of matter. Earth, water, Air, Fire;

- Earth = cube, Fire = tetrahedron, air = octahedron, water = icosahedron, universe = dodecahedron

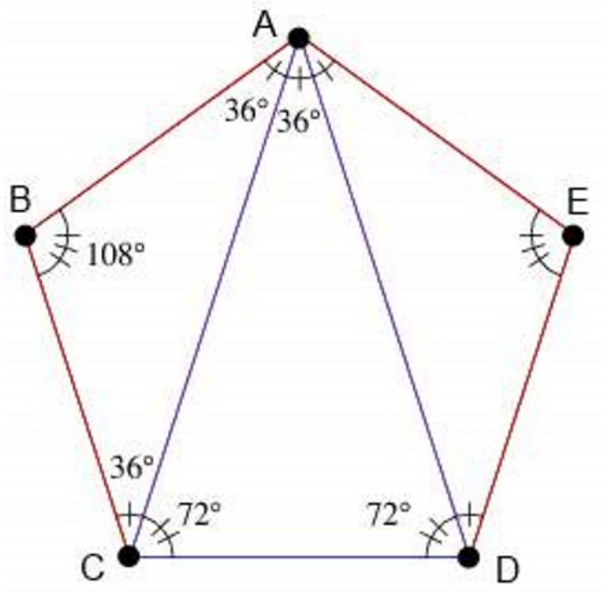
- Some posited the existence of a fifth element, a cosmic essence, the "quintessence" - taken to be the life force itself, that which animates the inanimate

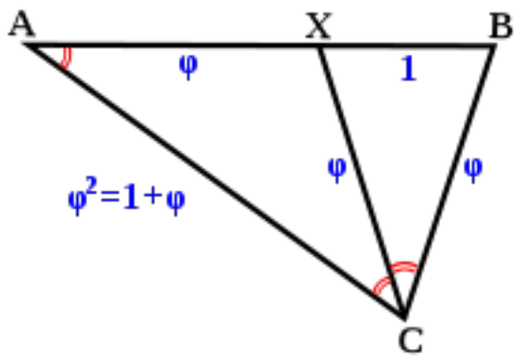
- Plato's Timaeus

- It is likely that the Greek interest in Phi started with their investigations of plane figures and solids

Euclid, around 300 BCE

- Euclid used the term "extreme and mean ratio"
- Elements, 13 volume work on geometry
- Golden triangle (36/72/72) and Golden Gnomons (36/36/72) - Golden Triangle has ratio of side to base of 1/phi - the two triangles on the side are the golden gnomons - unique quality that they can be dissected into smaller triangles that are also golden triangles and golden gnomons





Calculating Phi

- x/1 = x+1/x
- x^2 = 1+x
- x = 1 + 5^.5 / 2 = 1.618 or .618
- 1.618 ^2 = 2.618 and 1.618^.5 = .618

$$\Phi = \frac{1+\sqrt{5}}{2}$$

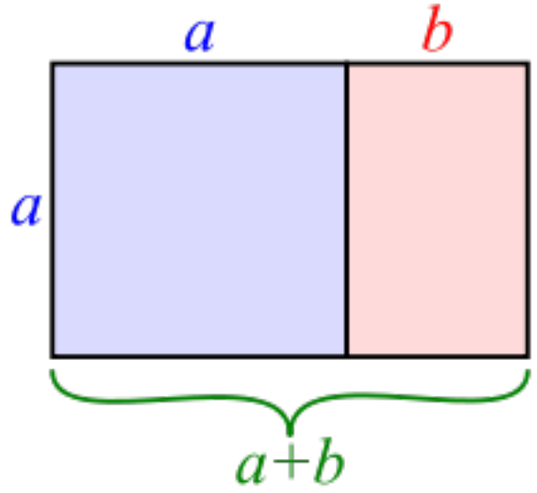
$$\Phi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}$$

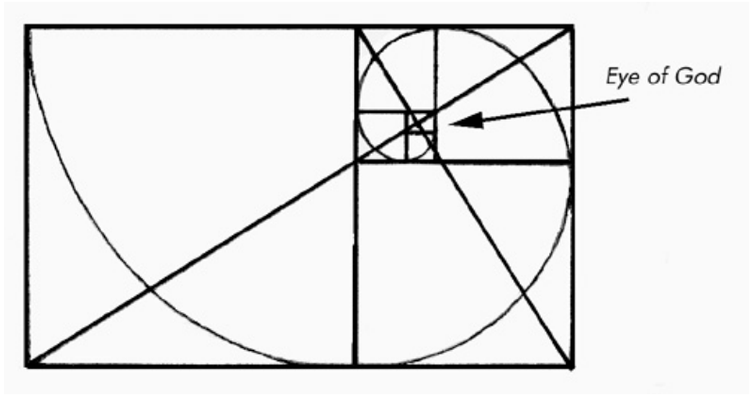
$$\Phi = 1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}}$$

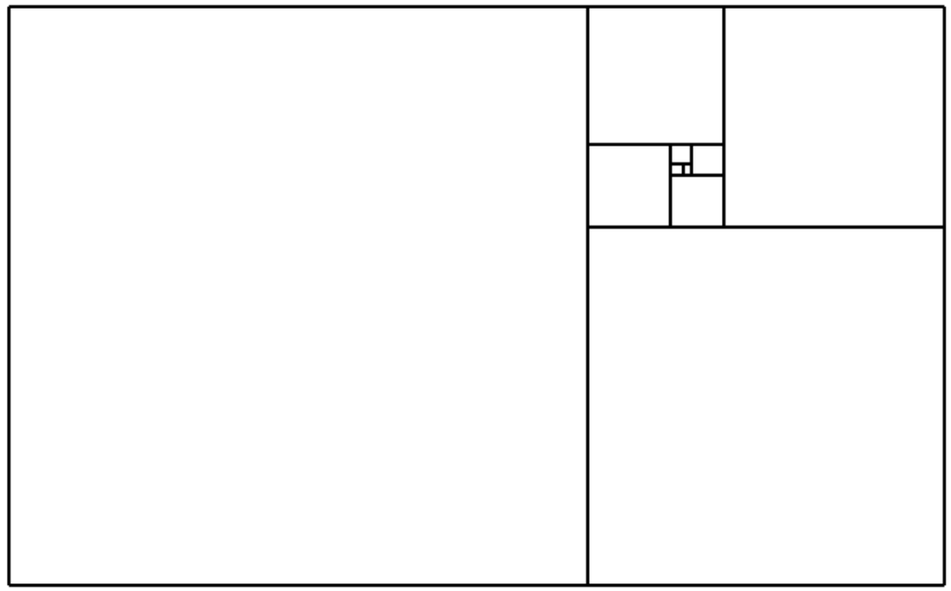
$$\frac{a+b}{a} = \frac{a}{b} = \phi$$

The Golden Rectangle

- The only rectangle that, cutting a square from it produces a similar rectangle.
- The point at which diagonals of two rectangles is the eye of god



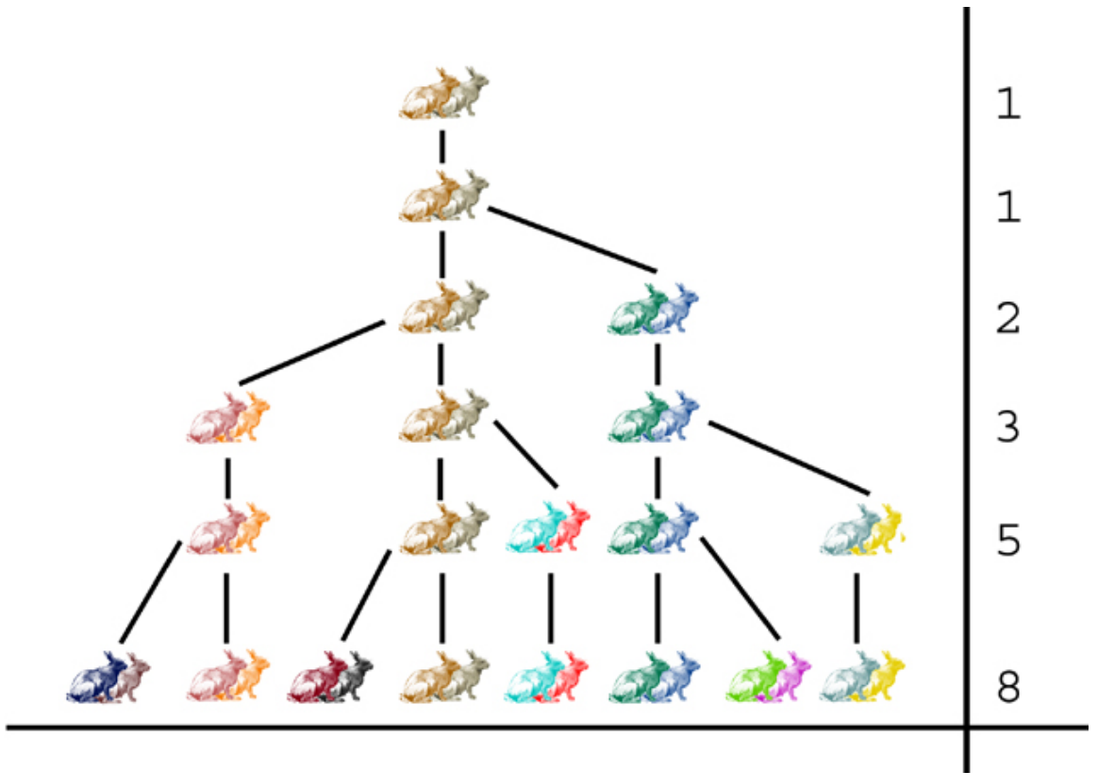




Fibonacci

- Liber abaci, published in 1202, advocated switching to the Hindu-Arabic system of numbers
- Fibonacci brought use of Golden ratio's properties to a higher level
- The rabbit problem demonstrates Fibonacci sequence
- There are many fascinating mathematical relations using phi

The further and further we go down in the Fibonacci sequence, the ratio of two successive numbers approaches phi. This property was discovered by Kepler.



Phyllotaxis and Phi in Nature